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L. I. Kudrazasher: More precise calculation of the coefficients of heat exchange between a gas and suspended particles by the application of the method of heat of boundary layer

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MORE PRECISE CALCULATION OF THE COEFFICIENTS OF HEAT EXCHANGE BETWEEN A GAS AND SUSPENDED PARTICLES BY THE APPLICATION OF THE METHOD OF HEAT OF BOUNDARY LAYER

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To study the process of evaporation of an atomized fluid (or dry atomized material) it is necessary, ~~in~~ as the first step course, to know the coefficient of heat transfer α from the gas to the particles of the atomized fluid or material. By defining the coefficient of heat exchange from the gas to ~~the~~ a body of spherical form in different ranges of variation of criteria of Reynolds is devoted a series of experimental works, of which note the following: Lyakhovski,[1] Loitzyanski and Schwab,[2] Virubov,[3] Chukhanov and Shapatma[4] and Früssling.[5] If the experimental works with respect to this question occupy evident place, then the problems of determining the coefficient of heat exchange from the gas to a spherical body is studied very little and in the literature there occurs a limited amount of theoretical investigation.

Boussinesq solved the problem of a cooling sphere with infinite heat conductivity in a gas stream having a velocity potential field, ~~by virtue~~ in view of which this work has only methodological interest, and no practical application.

The theory of Nusselt[6] deserves attention, for the heat exchange between a gas and particles of very small size, therefore its details are ~~omitted.~~

Let us denote

d particle diameter, meters

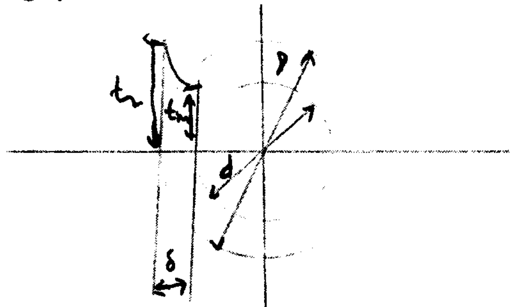
D diameter of the gas sphere surrounding the particle, meters

λ_G coefficient of heat conduction of the gas $\left(\frac{\text{K calor}}{\text{meter } ^\circ\text{C}}\right)$

t_G temperature of gas, $^\circ$C

t_M temperature of particle, $^\circ$C,

For particles whose diameter is less than 0.5 mm the temperature difference between the surface and the middle of the particle is negligibly small. The amount of heat, transmitted due to heat conduction through the spherical layer (see fig.) is determined from the well-known relation:



$$Q = \frac{2\pi\lambda_G(t_G - t_M)}{\frac{1}{d} - \frac{1}{D}} \quad (1)$$

On the other hand,

$$Q = \alpha\pi d^2(t_G - t_M) \quad (2)$$

Equating (1) and (2) we obtain the Nusselt criteria

$$Nu = \frac{\alpha d}{\lambda_G} = \frac{2}{1 - \frac{d}{D}}$$

The thickness of the gas sphere is

$$\delta = \frac{D-d}{2}$$

hence

$$\frac{d}{D} = \frac{1}{1 + \frac{2\delta}{d}}$$

Substituting in (3) we obtain

$$Nu = 2 + \frac{d}{\delta} \qquad (4)$$

Following Nusselt's notation, we will consider the size of the gas sphere enclosing the particle very large in comparison to the size of the particle itself, and therefore from equation (4) we obtain

$$Nu = 2 \qquad (5)$$

In other words, Nusselt, to calculate the coefficient of heat exchange from a gas to a particle of very small size, reduced the problem to the pure heat conduction through a half sphere whose outside diameter in the calculations was assumed large. This assumption may be considered correct only in the case when the relative velocity of the particle in the gas stream is practically zero. In other cases, application of formula (5) yields considerable error, which amount becomes extremely large when the Reynold's criteria is larger than one.

In the present paper is considered application of the method of heat of the boundary layer, introduced by G. N. Kruzhilin[7], to the more accurate Nusselt theory for the streamlines of a gas stream of a ball in the region of variation of Reynolds number from 0 to 100, characteristic for a dispersive system in a dry place, working with the scattering principle.

Möller,[8] on the basis of studies of streamlines of a sphere in a hydrodynamic regime, showed that for the value of Reynolds number Re = 100 the region of vortex formation in the neighborhood of a rear critical point is comparatively small. On the basis of this without large error we will assume, through the variation of Reynolds number from 0 to 100, that the streamlines of a sphere are without breaks.

As a subsequent ~~The following~~ assumption we assume ~~states~~ that the thickness of a hydrodynamic layer δ_1 near the surface of a sphere equals the thickness of heat of the boundary layer δ_2.

In view of the symmetry of the streamlines of a body the considerations are limited to the axi-symmetrical problem.

In the case of a body of revolution, the equation of heat balance for an element of heat of the boundary layer along $d\mathcal{S}$ leads to the integral relation.[9]

$$\frac{d}{ds}\left[\int_0^{\delta_2} w_S\left(1-\frac{t}{t_o}\right) z_1 dy\right] = \frac{a}{t_o}\left(\frac{\partial t}{\partial y}\right)_{y=0} z \qquad (6)$$

Here:

t temperature at a given point of the stream, reckoned from the temperature on the surface of the sphere

t_o temperature of the stream at a distance from the streamlines of the body

w_S projection of the velocity in the direction tangent to the generatrix of the body of revolution

a coefficient of temperature conduction of the gas (M^2/deg)

z_1 radius of an element of surface, normal to it

z radius of the body of revolution

The following relation exists between the radii z and z_1

$$z_1 = z + y \cos \theta$$

θ is the angle between the tangent and the generatrix of the body of revolution and the longitudinal axis of the body.

Inasmuch as for a sphere

$$\cos \theta = \cos\left(\frac{\pi}{2} - \varphi\right) = \sin \varphi$$

and also taking into account that

$$z = \frac{d}{2} \sin \varphi \qquad dx = \frac{d}{2}\, d\varphi$$

then we obtain

$$\frac{d}{d\varphi}\left[\sin \varphi \int_0^{\delta_2} w_s \left(1 - \frac{t}{t_o}\right)\left(1 + \frac{2y}{d}\right) dy\right] = \frac{d\,a}{2t_o}\left(\frac{\partial t}{\partial y}\right)_{y=0} \sin \varphi \tag{7}$$

or

$$\frac{d}{d\varphi}\left[\sin \varphi \frac{\delta_2^{\,2}}{d^2}\int_0^1 w_s \left(1 - \frac{t}{t_o}\right)\left(\frac{d}{\delta_2} + \frac{2y}{\delta_2}\right) d\left(\frac{y}{\delta_2}\right)\right] = \frac{a}{2t_o}\left(\frac{\partial t}{\partial y}\right)_{y=0} \sin \varphi \tag{8}$$

Here φ is the nondimensional angular coordinate on the generator of the sphere.

Let us assume that the field of temperature drop and velocity drop in the heat of the boundary layer is defined by the respective polynomials[10]

$$\frac{t}{t_o} = 2\left(\frac{y}{\delta_2}\right) - 2\left(\frac{y}{\delta_2}\right)^3 + \left(\frac{y}{\delta_2}\right)^4 \tag{9}$$

$$\frac{w_s}{w} = 2\left(\frac{y}{\delta_2}\right) - \left(\frac{y}{\delta_2}\right)^2 \tag{10}$$

where w is the stream velocity on the outer boundary of the heat of the boundary layer.

Having equations (9) and (10), with the aid of the integral relations (7) or (8) it is possible to calculate the thickness δ_2 of the heat of the boundary layer.

Usually in the solution of a similar problem, for simplification, the component $\left(\frac{2y}{\delta_2}\right)$ is neglected as compared to one. This assumption isn't possible in our case since in the region of the boundary studied, the variation of Reynolds number of thickness of the boundary layer is a quantity of the order of the radius of the sphere.

If this observation is taken into account, then we obtain from the integral relation (8) with the aid of (9) and (10):

$$\frac{\delta_2}{d}\,\frac{d}{d\varphi}\left[w\,\sin\varphi\;\frac{\delta_2^{\,2}}{d^2}\left(0.08 + 0.11\,\frac{d}{\delta_2}\right)\right] = \frac{a}{d}\;\sin\varphi \tag{11}$$

This equation belongs to a type of nonlinear differential equations with variable coefficients solvable by the approximate method of numerical integration.

Let us reduce by another simple method the integral equation (11), based on the law of sums: $0.08 + 0.11\,\frac{d}{\delta_2}$ factor equals $0.15\,\frac{d}{\delta_2}$. With ~~For~~ this rule the equation (11) becomes linear and easily integrable.

Actually, in the investigated region of variation of Reynolds number from 0 to 100 as the ratio $\frac{\delta_2^{\,2}}{d^2}$ varies from 0.5 to ∞; at the same time the quantity $\frac{d}{\delta_2}$ varies from 0 to 2. Therefore, the sum equal to

$0.08 + 0.11 \frac{d}{\delta_2}$ will vary in value from 0.08 to 0.30. Consequently without large error in the finite result it is possible to replace this sum by a factor $0.15 \frac{d}{\delta_2}$. Then the equation (11) becomes

$$\frac{\delta_2}{d} \frac{d}{d\varphi} \left[w \frac{\delta_2}{d} \sin \varphi \right] = 6.66 \frac{a}{d} \sin \varphi$$

Integrating, we obtain

$$\frac{\delta_2}{d} = \frac{f(\varphi)}{Pr^{1/2} \, Re^{1/2}} \qquad (12)$$

where

$$f(\varphi) = 3.65 \frac{W_0}{W_0 \sin_\varphi} \left[\int_0^\varphi \frac{W}{W_0} \sin^2 \varphi d\varphi \right]^{1/2} \qquad (13)$$

Here W_0 is the velocity of the stream at a distance from the streamline of the contour.

The value of the velocity on the outer boundary of the heat of the boundary layer may be determined from the following relation[11]

$$\frac{W}{W_0} = \frac{3}{2} \sin \varphi$$

Substituting in (13) we obtain

$$f(\varphi) = 2.98 \frac{\sqrt{\frac{2}{3} - \cos \varphi + \frac{\cos^3 \varphi}{3}}}{\sin^2 \varphi}$$

Therefore (12) yields

$$\frac{d}{\delta_2} = 0.336 \frac{\sin^2 \varphi}{\sqrt{\frac{2}{3} - \cos \varphi + \frac{\cos^3 \varphi}{3}}} \qquad (14)$$

The average value of the ratio $\left(\frac{\bar{d}}{\delta_2}\right)$ for the streamline of a sphere will be:

$$\left(\frac{\bar{d}}{\delta_2}\right) = \frac{\int_0^S \left(\frac{d}{\delta_2}\right) z\,ds}{\int_0^S z\,ds} = \frac{\int_0^\pi \left(\frac{d}{\delta_2}\right) \sin\varphi\,d\varphi}{\int_0^\pi \sin\varphi\,d\varphi}\, Pr^{1/2}\, Re^{1/2}$$

Substituting here (14) and integrating, we find:

$$\left(\frac{\bar{d}}{\delta_2}\right) = 0.388\, Pr^{1/2}\, Re^{1/2} \tag{15}$$

Since ~~with~~ by the condition $\delta = \delta_2$, then from (4) with the aid of (15) we finally obtain

$$\overline{Nu} = 2 + 0.388\, Pr^{1/2}\, Re^{1/2} \tag{16}$$

Assuming for air $Pr = 0.722$[12], we obtain

$$\overline{Nu} = 2 + 0.33\, Re^{1/2} \tag{17}$$

Comparison of the obtained equation (17) with experiments of Frössling is reproduced in the table. Here also is reproduced the Nusselt criteria, calculated by means of the Lykhovsky equation.[13]

Re	Calculated value of Nu: Frössling's Data	Lykhovsky's Data	Equation 17
0	2.000	2.290	2.000
1	2.250	2.290	2.330
10	2.790	3.344	3.040
50	3.770	4.880	4.330
100	4.500	6.680	5.300

CONCLUSIONS

From considering the table we may conclude:

1. Equation (17) gives values which are found between Frössling's data, the lower limit, and the Lyakovsky data representing the upper limit of the Nusselt criteria.

It is interesting to note that in the region of variation of Reynolds number from 10 to 100 equation (17) yields a result close to the arithmetic mean of the two opposite boundaries.

2. Neglect of the region of vortex formation in the neighborhood of the rear critical point, as expected, led to lower values of the Nusselt criteria in comparison to the data of Lyakovsky. Divergence in the results grows with increasing numerical value of Reynolds number.

In conclusion note the close coincidence of the calculated Nusselt criteria data by equation (17) with its calculation by the equation Klyachko obtained[14] starting from completely different assumptions. This equation has this form

$$\overline{Nu} = 2 + 0.16\, Re^{2/3} \tag{18}$$

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